EXGN Page 107

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1. $f(x) = x^2 + 4x - 5$

let $x = 0$ $\quad y = 0^2 + 4.0 - 5 = -5$

$(0, -5)$

let $y = 0$ $\quad x^2 + 4x - 5 = 0$

$(x-1)(x+5) = 0$

$x = 1$ or -5

$(1, 0)$ and $(-5, 0)$

$f(x) = x^2 + 4x - 5$

$f'(x) = 2x + 4$

At S.P's $\quad f'(x) = 0$

$2x + 4 = 0$

$2x = -4$

$x = -2.$

x	$-2-$	-2	$-2+$
$f(x)$	$-$	0	$+$
slope	$\backslash$	$-$	$/$

min at $(-2,$

$x = (-2, -9)$ $\quad y = (-2)^2 + 4(-2) -$

$= 4 - 8 - 5$

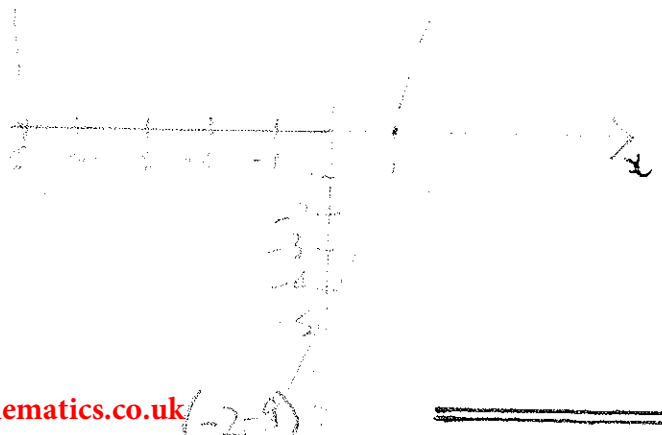
$= -13$

$= -9$

$x \to +\infty, \quad x^2 + 4x - 5 \xrightarrow{(x^2)} +\infty$

$x \to -\infty, \quad x^2 + 4x - 5 \to x^2 \to +\infty$

$y = x^2 + 4x - 5$



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2. $y = x(x-3)^2$

let $x=0$ $\quad y = 0(0-3)^2 = 0$

let $y=0$ $\quad x(x-3)^2 = 0$

$x = 0$ or 3

$x=0 \quad y = 0(0-3)^2 = 0 \quad (0,0)$

$x=3 \quad y = 3(3-3)^2 = 0 \quad (3,0)$

$y = x(x-3)^2$

$= x(x^2 - 6x + 9)$

$y = x^3 - 6x^2 + 9x$

$\frac{dy}{dx} = 3x^2 - 12x + 9$

At S.P.'s $\quad \frac{dy}{dx} = 0$

$3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$3(x-1)(x-3) = 0$

$x = 1$ or 3

$x=1 \quad y = 1(1-3)^2 = 1(-2)^2 = 1(4) = 4$

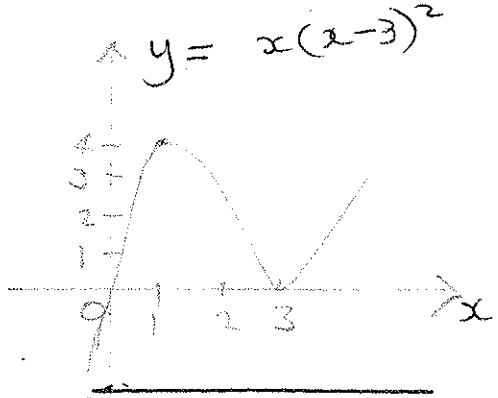
$x=3 \quad y = 3(3-3)^2 = 3(0)^2 = 0$

x	1_-	1	1_+	3_-	3	3_+
$f'x$	+	0	−	−	0	+
slope	/	—	\	\	—	/

Max at $(1,4)$ $\quad$ Min at $(3,0)$

$x \to +\infty, \quad x(x-3)^2 \to x^3 \to +\infty$

$x \to -\infty, \quad x(x-3)^2 \to x^3 \to -\infty$



3. $f(x) = x^3 - x^2$

let $x = 0$ $\quad y = 0^3 - 0^2 = 0$ $\quad (0,0)$

let $y = 0$ $\quad x^3 - x^2 = 0$

$x^2(x-1) = 0$

$x = 0$ or 1 $\quad (0,0)$ $(1,0)$

$x = 0$ $\quad f(0) = 0^3 - 0^2 = 0$

$x = 1$ $\quad f(1) =$

$f(x) = x^3 - x^2$

$f'(x) = 3x^2 - 2x$

At S.P.'s $\quad f'(x) = 0$

$3x^2 - 2x = 0$

$x(3x-2) = 0$

$x = 0$ or $2/3$.

$x = 0 \quad f(0) = 0^3 - 0^2 = 0$

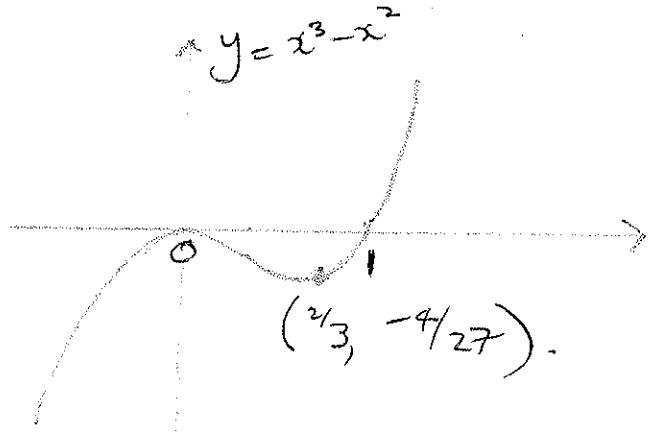
$x = 2/3 \quad f(2/3) = (2/3)^3 - (2/3)^2 = 8/27 - 4/9 = 8/27 - 12/27 = -4/27$

x	0_-	0_+	0_+	$2/3_-$	$2/3$	$2/3_+$
$f'(x)$	+	0	−	−	0	+
Slope	/	—	\	\	—	/

Max at $(0,0)$ $\quad$ Min at $(2/3, -4/27)$

$x \to +\infty, \; x^3 - x^2 \to x^3 \to +\infty$

$x \to -\infty, \; x^3 - x^2 \to x^3 \to -\infty$



4. $y = (x-2)(x+3)^2$

let $x = 0$ $\quad y = (0-2)(0+3)^2 = -2(3)^2 = -18$ $\quad (0, -18)$

let $y = 0$ $\quad (x-2)(x+3)^2 = 0 \quad x = 2$ or -3 $\quad (2, 0)$ $(-3, 0)$

$y = (x-2)(x+3)^2$

$= (x-2)(x^2+6x+9)$

$= x^3 + 6x^2 + 9x - 2x^2 - 12x - 18$

$y = x^3 + 4x^2 - 3x - 18$

$\frac{dy}{dx} = 3x^2 + 8x - 3$ $\quad$ At S.P's $\frac{dy}{dx} = 0$

$3x^2 + 8x - 3 = 0$

$(3x - 1)(x + 3) = 0$

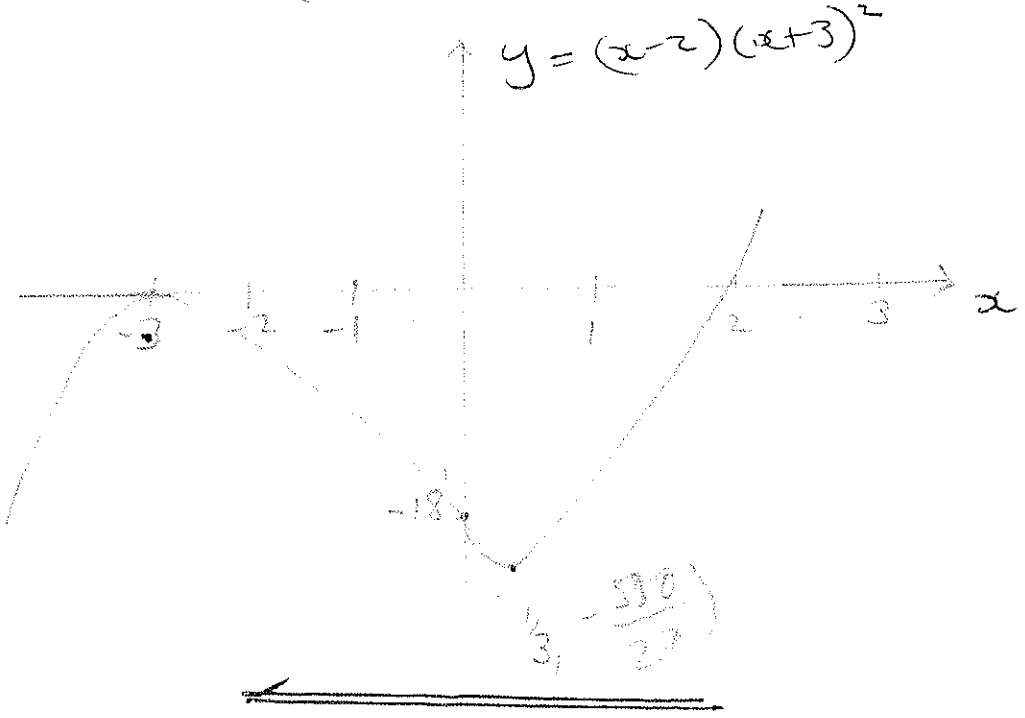
$x = \frac{1}{3}$ or -3.

$x = \frac{1}{3}$ $\quad y = (\frac{1}{3} - 2)(\frac{1}{3} + 3)^2 = (-\frac{5}{3})(\frac{10}{3})^2 = 25 \cdot -\frac{5}{3} \cdot \frac{100}{9} = -\frac{500}{27}$

$x = -3$ $\quad y = (-3-2)(-3+3)^2 = -5(0)^2 = 0$

x	$-3-$	-3	$-3+$	$\frac{1}{3}-$	$\frac{1}{3}$	$\frac{1}{3}+$
$\frac{dy}{dx}$	+	0	−	−	0	+
slope	/	—	\	\	—	/

Max T.P at $(-3, 0)$ $\quad$ Min T.P at $(\frac{1}{3}, -\frac{500}{27})$



5. $y = 8 + 2x - x^4$

let $x = 0$ $\quad y = 8 + 2(0) - 0^4 = 8$ $\quad (0, 8)$

let $y = 0$ $\quad 8 + 2x - x^4 = 0$

$(4 - x^2)(2 + x^2) = 0$ (ignore)

$x^2 = 4$

$x = \pm 2$. $\quad (-2, 0)\ (2, 0)$

$y = 8 + 2x - x^4$

$\frac{dy}{dx} = 2 - 4x^3$

At SP. $\quad 2 - 4x^3 = 0$

$4x^3 = 2$

$x^3 = \frac{1}{2}$

$x = \sqrt[3]{\frac{1}{2}}$

7. $f(x) = 2x^2 - x^4$

let $x = 0$ $\quad f(0) = 0$ $\quad (0,0)$

let $y = 0$ $\quad 2x^2 - x^4 = 0$

$x^2(2 - x^2) = 0$ $\quad (0,0)$

$x = 0$ or $\pm\sqrt{2}$ $\quad (\sqrt{2}, 0)$ $\quad (-\sqrt{2}, 0)$

$f(x) = 2x^2 - x^4$

$f'(x) = 4x - 4x^3$ $\quad$ At S.P.'s $f'(x) = 0$

$4x - 4x^3 = 0$

$4x(1 - x^2) = 0$

$x = 0, 1, -1.$

$x = 0$ $\quad f(0) = 0$ $\quad (0,0)$

$x = 1$ $\quad f(1) = 2.1^2 - 1^4 = 2 - 1 = -1$ $\quad (1, -1)$

$x = -1$ $\quad f(-1) = 2(-1)^2 - (-1)^4 = 2 - 1 = 1$ $\quad (-1, 1)$